

② $I = \iint_S 8xy \, dx \, dy$, $x=0$, $y=x$, $y=1-x$, $y=2-x$

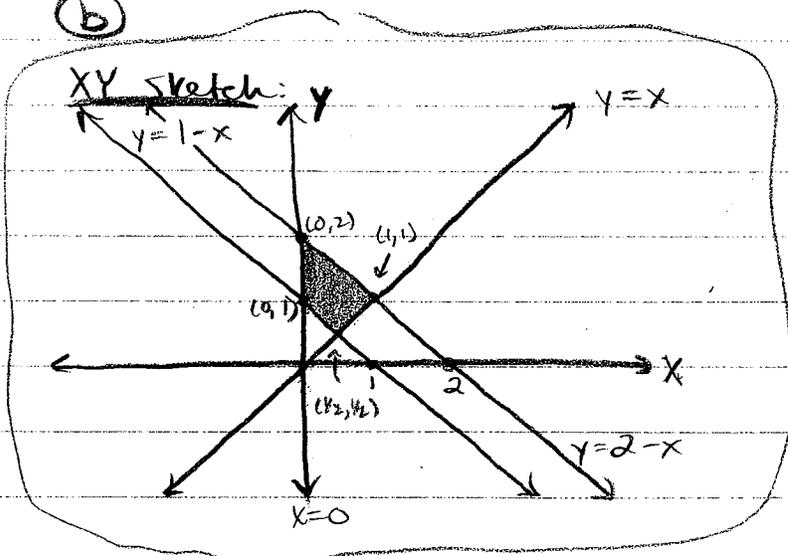
① $u = x+y$, $v = x-y$

$x = u-y \Rightarrow v = u-y-y \Rightarrow v = u-2y \Rightarrow v-u = -2y$

$y = \frac{u-v}{2}$

$x = u - \left(\frac{u-v}{2}\right) = \frac{2u}{2} - \frac{u}{2} + \frac{v}{2} = \frac{u}{2} + \frac{v}{2} = \frac{u+v}{2} \Rightarrow x = \frac{u+v}{2}$

③



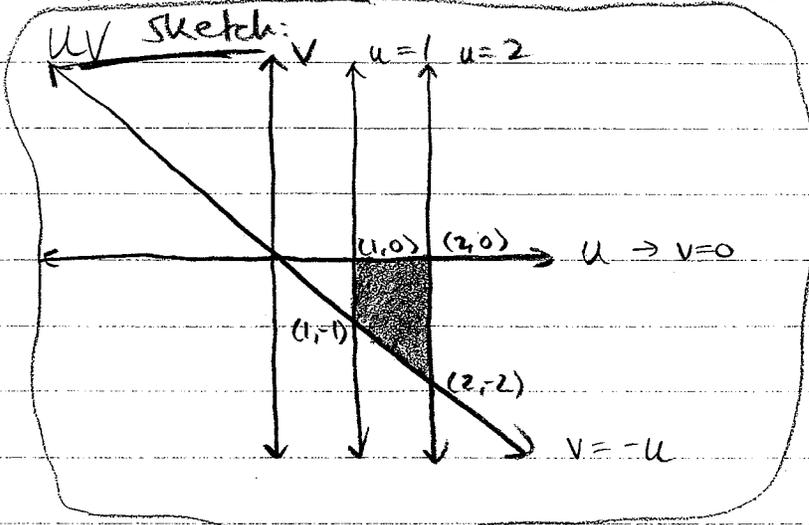
$x=0 \Rightarrow \frac{u}{2} + \frac{v}{2} = 0 \Rightarrow u+v=0 \Rightarrow u=-v$ or $v=-u$

$y=x \Rightarrow \frac{u}{2} - \frac{v}{2} = \frac{u}{2} + \frac{v}{2} \Rightarrow -v=v \Rightarrow v=0$

$y=1-x \Rightarrow \frac{u}{2} - \frac{v}{2} = 1 - \frac{u}{2} - \frac{v}{2} \Rightarrow u=1$

$y=2-x \Rightarrow \frac{u}{2} - \frac{v}{2} = 2 - \frac{u}{2} - \frac{v}{2} \Rightarrow u=2$

UV Sketch:



$$\textcircled{c} \quad J(u, v) = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} = \frac{1}{4} - \frac{1}{4} = -\frac{1}{2}$$

$$\begin{aligned} & \int_1^2 \int_0^{-u} -\frac{1}{2}(8) \left(\frac{u+v}{2}\right) \left(\frac{u-v}{2}\right) dv du \\ &= - \int_1^2 \int_0^{-u} (u+v)(u-v) dv du \\ &= - \int_1^2 \int_0^{-u} (u^2 + uv - uv - v^2) dv du \\ &= - \int_1^2 \int_0^{-u} (u^2 - v^2) dv du \\ &= \boxed{\int_1^2 \int_0^{-u} (v^2 - u^2) dv du} \end{aligned}$$

$$\begin{aligned} \textcircled{d} \quad & \int_1^2 \left[\frac{v^3}{3} - vu^2 \right]_0^{-u} du = \int_1^2 \left[\frac{u^3}{3} + u^3 - 0 - 0 \right] du \\ &= \int_1^2 \left[\frac{2}{3}u^3 \right] du = \frac{2}{3} \left[\frac{u^4}{4} \right]_1^2 \\ &= \frac{2}{3} \left[\frac{1}{4} \right] (16 - 1) = \frac{2 \cdot 1 \cdot 15}{3 \cdot 4 \cdot 2} = \boxed{\frac{5}{2}} \end{aligned}$$

