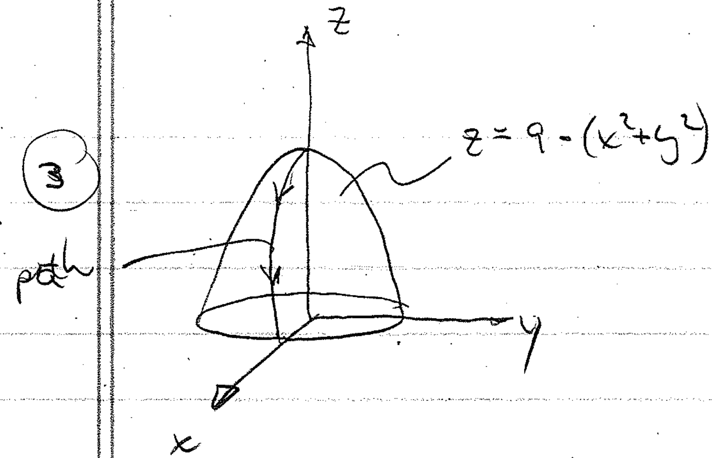
3



a) $\underline{r}(t) = t\,\hat{\imath} + 0\,\hat{\jmath} + f(t)\,\hat{k}$ where $x(t), y(t)$ & $z(t)$ must satisfy

$\uparrow$ $x(t)$ $\quad \uparrow$ $y(t)$ $\quad \uparrow$ $z(t)$

$$z = 9 - x^2 - y^2$$

$$\therefore \; z(t) = 9 - t^2 - 0^2 = f(t)$$

so $\underline{r}(t) = t\,\hat{\imath} + 0\,\hat{\jmath} + (9 - t^2)\,\hat{k}$ $\Leftarrow$

$\underline{v}(t) = \hat{\imath} + 0\,\hat{\jmath} + (-2t)\,\hat{k}$

b) Time when $z(t) = 9 - t^2 = 0 \quad \Rightarrow \quad t = 3$ $\Leftarrow$

c) $s = \displaystyle\int_{t=0}^{3} |\underline{v}|^2\,dt = \int_{t=0}^{3} \sqrt{1 + 4t^2}\,dt \qquad u = 2t \quad du = 2\,dt$

$$= \frac{1}{2}\int_{u=0}^{u=6} \sqrt{1 + u^2}\,du$$

so

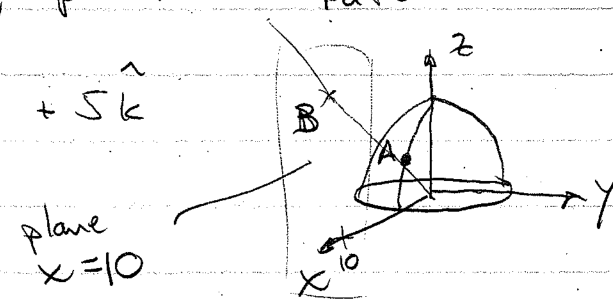
$$2s = \left[\frac{u}{2}\sqrt{1+u^2} + \frac{1}{2}\ln\left(u + \sqrt{1+u^2}\right)\right]_{u=0}^{6}$$

or

$$s = \frac{1}{4}\left[6\sqrt{37} + \ln\left(6 + \sqrt{37}\right)\right] \Leftarrow$$

d) at time $t = 2$, pt A on path is

$$\underline{r}(2) = 2\,\hat{\imath} + 0\,\hat{\jmath} + 5\,\hat{k}$$



(3) cont.

So line through origin + A is

$$\underline{L}(s) = 2s\hat{i} + 0\hat{j} + 5s\hat{k}$$

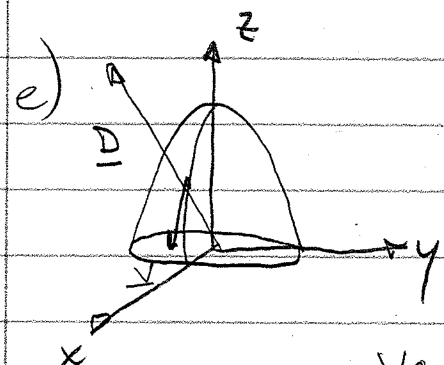
which has direction $\underline{D} = 2\hat{i} + 0\hat{j} + 5\hat{k}$

To find s when $\underline{L}$ intersects plane $x = 10 = 2s$

so $s = 5$

Thus coord. of B are $\underline{L}(5) = 10\hat{i} + 0\hat{j} + 25\hat{k}$

or $(10, 0, 25)$ ←

e) 

Want ∡ between $\underline{V}(t=2) = \hat{i} - 4\tilde{k}$

and $\underline{D} = 2\hat{i} + 0\hat{j} + 5\tilde{k}$

$$\underline{V} \circ \underline{D} = -18 = |\underline{V}||\underline{D}| \cos\theta$$

$$\cos\theta = \frac{-18}{|\underline{V}||\underline{D}|} = \frac{-18}{\sqrt{17}\sqrt{29}}$$

$$\theta = \cos^{-1}\left(\frac{-18}{\sqrt{17}\sqrt{29}}\right)$$ ←